# Homework 10

**P9.1.2** The terminal of one coil in Figure P9.1.2 is marked with a dot. (a) Mark one terminal of the other coils with a dot; (b) connect the coils in series for maximum total inductance; (c) determine the inductances of coils 2 and 3, assuming $L_1$ = 40 mH; $M_{12}$ = $M_{23}$ = 20 mH, and $k_{12}$ = $k_{23}$ = $1/\sqrt{2}$ ; (d) determine $M_{13}$, assuming $k_{13}$ = 0.5.

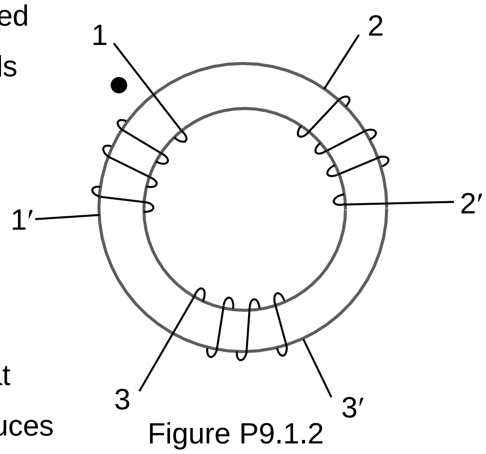

Figure P9.1.2

**Solution:** (a) The dots on the windings should be such that current entering at the dotted terminals produces flux in the same direction in the core.

(b) To produce maximum inductance, the coils have to be connected so that fluxes are additive.

(c) $M_{12} = k_{12}\sqrt{L_1L_2}$ , or, $20 = \sqrt{40L_2}\,/\sqrt{2}$ , which gives $L_2$ = 800/40 = 20 mH; $20 = \sqrt{20L_3}\,/\sqrt{2}$ , which gives $L_3$ = 800/20 = 40 mH;

$M_{13} = 0.5\sqrt{40\times 40} = 20$ mH.

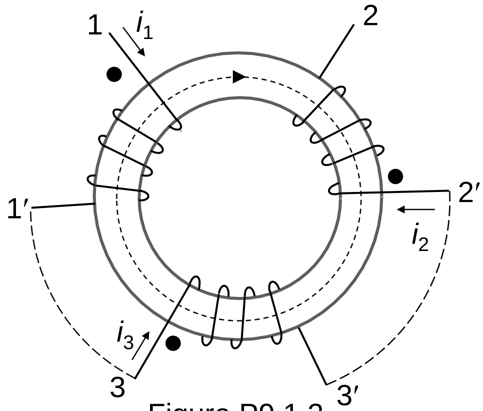

Figure P9.1.2

**P9.1.20** Determine $v_1(t)$ and $v_2(t)$ in Figure P9.1.20, assuming $i_1(t)$ = (64$t$ + 50) A and $i_2(t)$ = 15$t$ A.

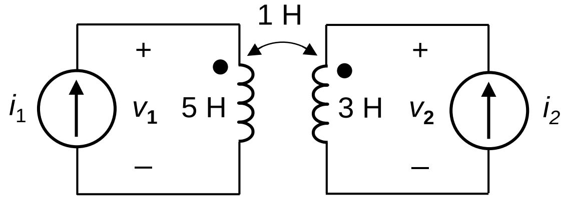

Figure P9.1.20

**Solution:** $v_1 = L_1di_1/dt + Mdi_2/dt = 5\times64 + 15$ = 335 V; $v_2 = L_2di_2/dt + Mdi_1/dt = 3\times15 + 64$ = 109 V.

**P9.2.3** Determine $Z_{in}$ in Figure P9.2.3.

**Solution:** The impedance encountered by the current **I** is: $j\omega L_1 + j\omega L_2 - j2\omega M - j/\omega C = j20 + j20 - j20 - j20 = 0$. Hence, $Z_{in} = 0$.

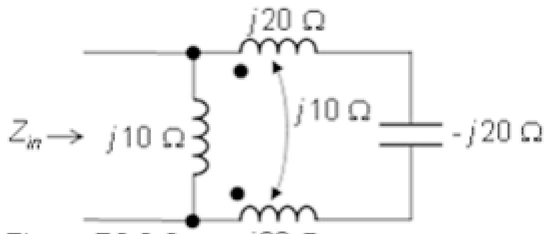


Figure P9.2.3

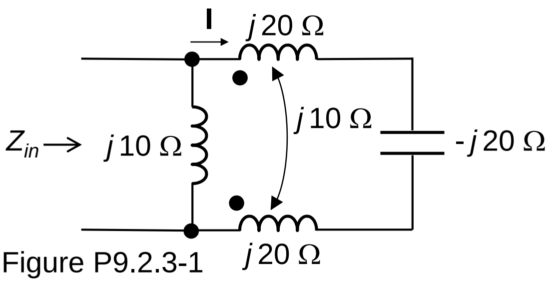


Figure P9.2.3-1

**P9.2.7** By substituting the T-equivalent circuit of either Figure 9.4.1b or Figure 9.4.1d for the linear transformer in Figure P9.2.7, show that the input impedance $\mathbf{V_1}/\mathbf{I_1}$ can be expressed as: $Z_{in} = \dfrac{\mathbf{V_1}}{\mathbf{I_1}} = j\omega L_1 + \dfrac{\omega^2 M^2}{j\omega L_2 + Z_L}$.

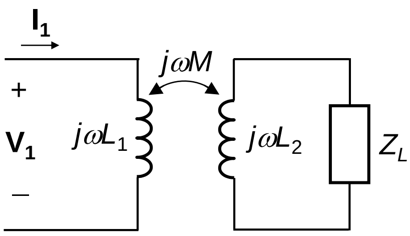


Figure P9.2.7

Note that because $M$ is squared in this expression, $Z_{in}$ is independent of the dot markings.

**Solution:** Assuming the dot markings of Figure 9.4.1a,

$$Z_{in} = j\omega(L_1 - M) + \frac{j\omega M\left(j\omega(L_2 - M) + Z_L\right)}{j\omega L_2 + Z_L} =$$

$$j\omega L_1 + \frac{\omega^2 M L_2 - j\omega M Z_L - \omega^2 M L_2 + \omega^2 M^2 + j\omega M Z_L}{j\omega L_2 + Z_L}$$

$$= j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + Z_L}$$ Since $M$ is squared, this result is independent of the dot markings.

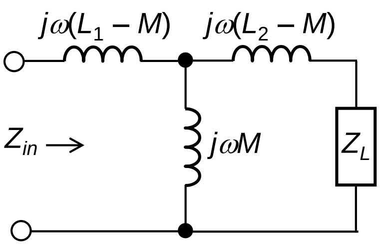


Figure P9.2.7-1

**P9.2.11** Using the result of Problem P9.2.7, determine $k$ in Figure P9.2.10 so that: (a) the input impedance $Z_{in}$ is purely resistive; (b) $\mathbf{I_1}$ is maximum.

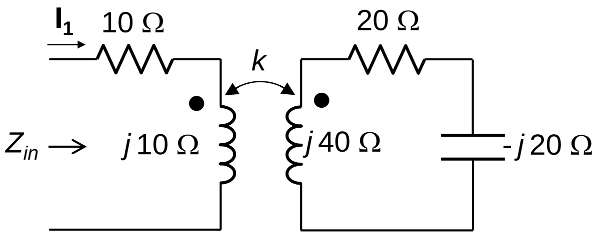


Figure P9.2.11

**Solution:** Input impedance = 10 + $j$10 + $\dfrac{10\times 40k^2}{j40+20-j20}$ = 10 + $j$10 + $\dfrac{400k^2}{20+j20}$ = 10 + $j$10 + $\dfrac{20k^2(1-j)}{2}$ = 10(1+ $k^2$) + $j$10(1 – $k^2$).

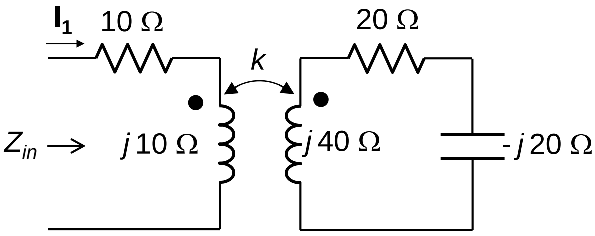


Figure P9.2.11

(a) The impedance is purely resistive when $k = 1$.

(b) $|Z_{in}|^2 = 100[(1+ k^2)^2 + (1 – k^2)^2]$; current is maximum when bracketed quantity is minimum. Bracketed quantity is: 2 + 2$k^4$ and is a minimum when $k = 0$.

**P9.2.18** Determine the stored energy in the circuit of Figure P9.2.18 in the dc steady state, assuming $M$ = 1 H.

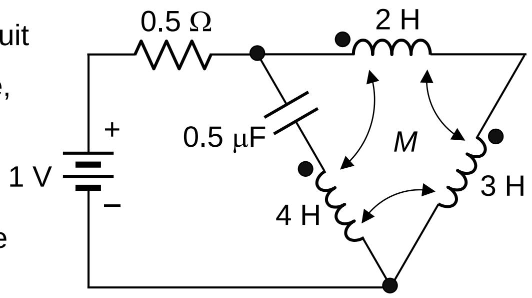


Figure P9.2.18

**Solution:** No current flows in the dc state through the 4 H inductor because of the series capacitor. $L_{eq}$ of the 2 H and 3 H inductors is 2 + 3 + 2×1 = 7 H. This inductor acts as a short circuit, so that the current through it is 1/0.5 = 2 A, and the voltage the capacitor is zero. The energy stored in the circuit is 0.5×7×4 = 14 J.

**P9.2.27** **P9.2.27** Derive TEC between terminals ‘ab’ in Figure P9.2.27.

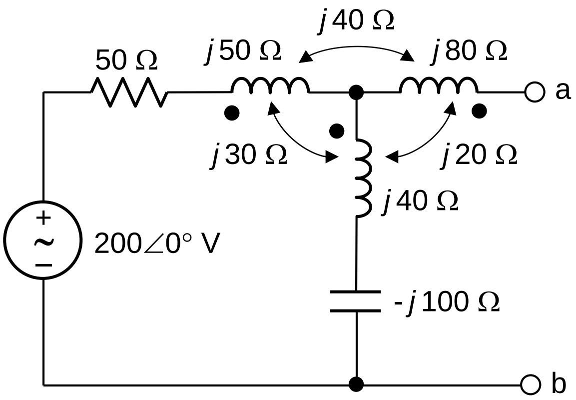


Figure P9.2.27

**Solution:** With terminals ab open circuited, the equivalent inductance in mesh 1 is $j50 + j40 + 2\times j30 = j150\ \Omega$. Hence,

$\mathbf{I}_1 = \dfrac{200}{50 + j150 - j100} = \dfrac{4}{1+j} = 2(1-j)$ A. $\mathbf{V}_{Th} =$ $(j40 - j100 + j40 + j30 + j20)\mathbf{I}_1 = j60(1 - j) = 60(1 + j)$ V.

With terminals ab short circuited, $200 = (50 + j50 + j40 - j100)\mathbf{I}_1 - (j40 - j100)\mathbf{I}_2 + j30\mathbf{I}_1 + j30(\mathbf{I}_1 - \mathbf{I}_2) - j40\mathbf{I}_2 - j20\mathbf{I}_2$, or $(50 + j50)\mathbf{I}_1 - j30\mathbf{I}_2 = 200$. For mesh 2: $(j80 + j40 - j100)\mathbf{I}_2 - (j40 - j100)\mathbf{I}_1 + j20\mathbf{I}_2 + j20(\mathbf{I}_2 - \mathbf{I}_1) - j30\mathbf{I}_1 - j40\mathbf{I}_1 = 0$, or $-j30\mathbf{I}_1 + j60\mathbf{I}_2 = 0$, or $\mathbf{I}_1 = 2\mathbf{I}_2$.

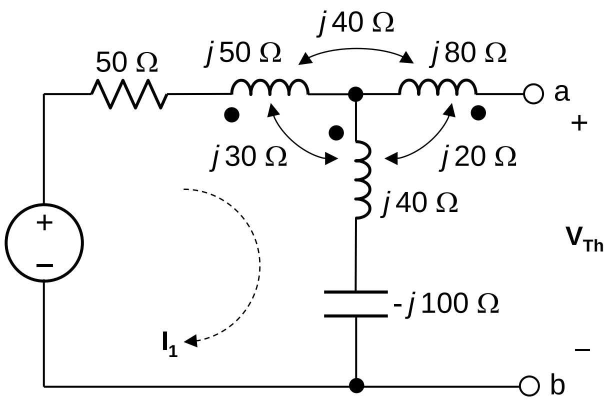


Figure P9.2.27-1

Substituting, $\mathbf{I}_{sc} = \mathbf{I}_2 = \dfrac{20}{10 + j7}$ A.

$Z_{Th} = \dfrac{60(1+j)(10+j7)}{20} = 3(3 + j17)\ \Omega$.

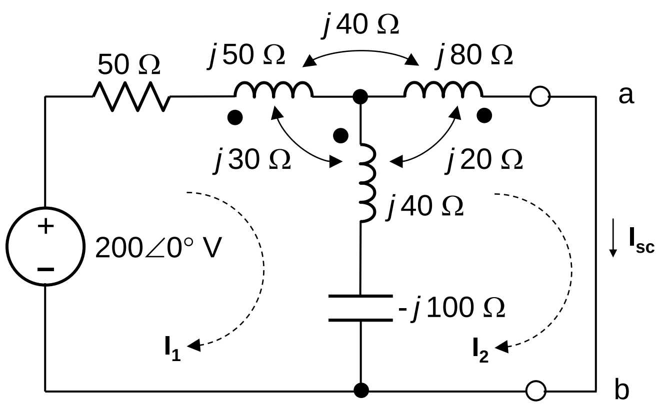


Figure P9.2.27-2

**P9.2.38** For the circuit of Figure P9.2.38, (a) Derive the mesh-current equations; (b) determine $\mathbf{V}_{ab}$.

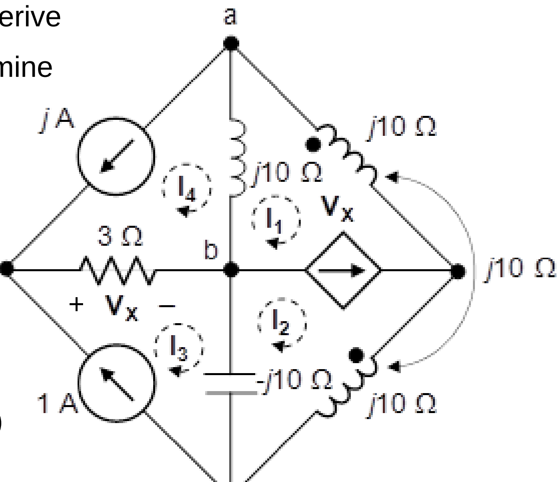

Figure P9.2.38

**P9.2.38** The mesh-current equations are:

Mesh 1: $j20\mathbf{I}_1 + j10\mathbf{I}_2 - j10\mathbf{I}_4 = \mathbf{V}_Y$, where $\mathbf{V}_Y$ is a voltage rise across the dependent source in the direction of $\mathbf{I}_1$

Mesh 2: $j10\mathbf{I}_1 + (j10 - j10)\mathbf{I}_2 + - (-j10\mathbf{I}_3)$
$= -\mathbf{V}_Y$

Adding these two equations,

$$j30\mathbf{I}_1 + j10\mathbf{I}_2 + j10\mathbf{I}_3 - j10\mathbf{I}_4 = 0$$

Substituting $\mathbf{I}_4 = -j$ and $\mathbf{I}_3 = 1$ and dividing by $j10$ gives: $3\mathbf{I}_1 + \mathbf{I}_2 = -1 - j$. Moreover, $\mathbf{V}_X = \mathbf{I}_2 - \mathbf{I}_1$, where $\mathbf{V}_X = 3(1 + j)$. Substituting for $\mathbf{I}_2$: $-4\mathbf{I}_1 = (1 + j) + 3(1 + j) = 4(1 + j)$, or $-\mathbf{I}_1 = (1 + j)$. $\mathbf{V}_{ab} = j10(\mathbf{I}_4 - \mathbf{I}_1) = j10(-j + 1 + j) = j10$ V.

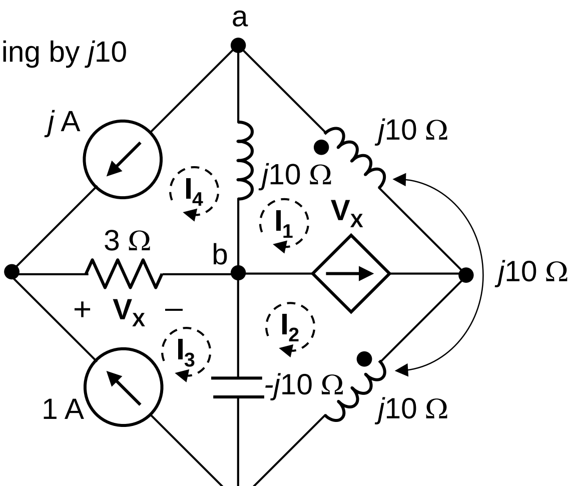

Figure P9.2.38